Principles and Practice

Examination Review

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Strength of Materials
Structural Analysis
Overview

- In strength of materials, we are looking for basic stresses induced in structural members due to loading, and the resulting deflections.

- Using derived equations, a close approximation of the stress level and deflections in the member can be found and compared with allowed values.
Outline (continued)

- Deflections
  - Axial deformations
  - Torsional deformations
  - Beam deflections
Direct Stresses

- **Tension and Compression stresses**
  - where $\sigma = \frac{P}{A}$ - stretching and squashing parallel to load and long axis of member.

- **Shear stresses**
  - where $\tau = \frac{P}{A}$ - shear stresses - wiping stresses - cross-shear on bolts, stresses in web of beam - stresses across the long axis of the member.
Torsional Stresses

\[ \sigma = \frac{T r}{J} \]

- where \( \tau \) is the shear stress
- \( T \) is the applied torque
- \( r \) is the outer radius of the bar
- \( J = \pi (r_{outer}^4 - r_{inner}^4) / 2 \)
- Shear stress is across the end face of the bar, a maximum on the outside fiber, and zero at the center of the bar. The equation applies only to circular rods and pipes.
Shear and Bending Moment Diagrams

- Use statics to solve for reactions.
- The area under any diagram gives the change in value on the next diagram.
- The value on any diagram gives the slope of the next diagram.
- If load is uniform constant, \( x_{\text{bar}} = \) the starting shear (from shear diagram) / the load rate (from load diagram.)
Loaded beam

$w = 3 \text{ kips/ft}$

$w = 2 \text{ kips/ft}$

$6 \text{kips}$

$6'$

$7'$

$5'$

$R_B = 25 \text{kips}$

$(a)$

$R_D = 14 \text{kips}$
Bending Stresses

\[ \sigma = \frac{Mc}{I} \]

- where \( \sigma \) is the bending stress in the beam
- \( M \) is the applied moment
- \( c \) is the distance from the neutral axis to an outside fiber on the beam
- \( I \) is the moment of inertia of the beam
- Stresses are normal (tensile or compressive) and are zero at the neutral axis and maximum on the outside fibers of the beam.
Beam Cross Shearing Stresses

\[ \tau = \frac{VQ}{It} \text{ where} \]

- \( V = \text{Shear force from shear diagram} \)
- \( Q = \text{First moment of area above level where shear stress is desired} \)
- \( I = \text{Moment of inertia about NA} \)
- \( t = \text{thickness of beam at level where shearing stresses are desired} \)
Cylindrical Pressure Vessels

- Hoop stresses act around the tank, in tension - $\sigma_c = \frac{pr}{t}$
- Longitudinal or axial stresses act along the axis of the tank - $\sigma_l = \frac{pr}{2t}$
- Maximum shear stresses - $\tau = \frac{pr}{2t}$
Spherical Pressure Vessels

- All tensile stresses in all directions
  \[ \sigma_c = \frac{pr}{2t} \]
- Max. shear stresses
  \[ \tau = \frac{pr}{4t} \]
Combined Stresses

- Axial and torsion (tension and shear)
- Axial and bending (tension and tension)
- Stresses under footings (compression and compression)
- Axial and torsion and bending and pressure (cheee!)
- Plane stress equations for combined stresses
Axial and Torsional Stresses

- Compute axial stresses and torsional stresses separately and put them on the stress block. Determine principal stresses from principal stress equations (see later.)
Columns

- Pcr is the critical failure buckling load
- E is the modulus of elasticity
- I is the moment of inertia
- L is the length between points of inflection
- K is the effective length factor

Pinned on each end - $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$ with no factor of safety.
Deflections of Axially Loaded Members

\[ \delta_{\text{load}} = \frac{PL}{AE} \text{ where} \]
- \( P \) is the axial load
- \( L \) is the length of the member
- \( A \) is the cross-sectional area
- \( E \) is the modulus of elasticity

\[ \delta_{\text{temp}} = \alpha \Delta T L \text{ where} \]
- the coefficient of thermal expansion is \( \alpha \)
- and \( \Delta T \) is the change in temperature
\[ E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta/L} = \frac{P}{A\Delta} \quad \text{or} \quad \Delta = \frac{PL}{AE} \]
Deflections of Torsionally Loaded Circular Shafts

\[ \theta = \frac{TL}{GJ} = \text{rotation between any two points on the shaft, and} \]

- \( T = \text{the applied torque} \)
- \( L = \text{the length of the shaft} \)
- \( G = \text{the shear modulus of elasticity} \)
- \( J = \text{the polar moment of inertia of the shaft} = \pi \left( r_{outer}^4 - r_{inner}^4 \right) / 2 \)
\[ \theta = \frac{T_L}{C} \]
Deflections of beams

- Direct integration
- Singularity functions
- Moment area
  - Tables and superposition - see especially the AISC codes.
\[
\sigma = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta
\]

\[
\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta
\]

\[
\tan 2\theta_p = \frac{-\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}
\]

\[
\sigma_{\text{max}} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}
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\sigma_{\text{min}} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}
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\[
\tau_{\text{max}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}
\]
MOMENT DISTRIBUTION

1) Divide the frame into individual MEMBERS and JOINTS.

2) RELATIVE STIFFNESS FACTORS - \(K_{ij}\)

Compute the RELATIVE STIFFNESS FACTOR, \(K_{ij}\) for each end of each member. The relative stiffness factors give you a measure of how stiff each member is, and thus how much of a moment it will resist, since stiffer members resist higher moments.

For member II, standing at point I on the member and looking toward the other end J:

\[ K_{ij} = 3 E_u I_u/L_u \text{ if the other end of member II is pinned,} \]

\[ K_{ij} = 4 E_u I_u/L_u \text{ if the other end of member II is fixed, or connected to other members.} \]

If the modulus of elasticity of the member E is constant throughout the frame, you may set \(E = 1\), as it will cancel out. The same applies to the moment of inertia \(I\) - if I is constant throughout the frame, set \(I = 1\). Also, although unlikely, if all member lengths were the same, you may use \(L = 1\). This is acceptable, since the moments are distributed to the members in proportion to their RELATIVE stiffness, rather than their absolute stiffness.

Enter these \(K_{ij}\)’s into your moment distribution table.

3) DISTRIBUTION FACTORS - \(DF_{ij}\)

Compute the DISTRIBUTION FACTOR, \(DF_{ij}\) at each end of each member. The distribution factors tell you how much moment will "fan out" from a joint, should you apply an unbalanced moment on that joint.

For member II, standing at point I on the member and looking towards the other end J:

\[ DF_{ij} = \text{negative} \ K_{ij}/\sum K_{ij} \text{ of all members entering into joint I} \]

Where \(\sum K_{ij} \text{ of all members entering into joint I}\) is the sum of the \(K_{ij}\) for all members coming into joint I.

Be sure to enter the distribution factors into your table as negatives - it will save you many errors. Enter these DF’s into your moment distribution table.

4) CARRY-OVER FACTORS - \(COF_{ij}\)

Compute the CARRY-OVER FACTOR - \(COF_{ij}\) for each end of each member. The carry-over factors tell you how much moment "fanning out" from joint I, through member II, will make it to the other end of member II. This carry over moment will now unbalance the moment at joint J.
COF$_u = 0$, if the far end of member $IJ$ is pinned,

COF$_u = 1/2$ if the far end of member $IJ$ is fixed, or connected to other members.

Enter these COF's into your moment distribution table.

5) FIXED END MOMENTS - FEM$_u$

Assume that every joint in the frame is fixed and compute the fixed end moments FEM$_u$ at each end of each member. The fixed end moments are probably better than nothing, just as they are. The only problem is that if the FEM to the left of a joint is +100 kft, and the FEM on the right of the joint is -20 kft, the joint is out of balance by +80 kft. Thus some poor guy is going to have to stand there with a wrench for the next 50 years and hold on with +80 kft to keep the joint from rotating.

Note that fixed end moments have a rigid sign convention: if the FEM on the end of the member is clockwise, it is positive, and if counterclockwise, negative. Period. No exceptions. Thus it doesn't matter if the member is a horizontal beam, or a vertical column, or a slanted member, or if you number the members from top to bottom, or right to left, the sign convention is the same.

Values of FEM are found in any structural analysis books, and in our review book.

Enter these FEM's into your moment distribution table.

6) OUT OF BALANCE MOMENTS - first OOB$_i$

$$OOB_i = \sum \text{FEM of all members entering into joint } I$$

Go to each joint and see how badly the moments are out of balance, by summing up all the FEM's coming into that joint. Simply add up the FEM values from your table, plus or minus, and write this sum, with its sign, in the OOB$_i$ (out of balance for joint $I$) box. Note that the OOB$_i$ has only one subscript, since it relates to the joint, rather than to the member moments.

7) DISTRIBUTIVE MOMENTS - first DIST$_u$

These out of balance moments must be released, and when released will distribute into adjacent members by the equation

$$\text{DIST}_u = \text{OOB}_i \times \text{DF}_u$$

You can determine how much each member around joint $I$ will take by multiplying the OOB$_i$ moment by the adjacent member's distribution factor DF$_u$. Don't forget to use a negative for all DF$_u$ values, since this takes care of the fact that the out of balance moment, when released, is actually of opposite sign than the sum of the fixed end moments. Write these distributed moments in the table.
8) CARRY OVER MOMENTS - first COM$_{II}$

$$\text{COM}_{II} = \text{DIST}_{II} \times \text{COF}_{II}$$

These distributed moments will immediately carry over to the other ends of the members, in proportion to the member carry over factors. Thus if $C_{II}$ is 1/2, then 1/2 of the moment distributed into member II will carry over to the far end of the member. The sign remains the same. If $C_{II}$ is 0, no moment will carry over to the far end of the member II. Write these carry over moments (COM$_{II}$) into the table at the other end of the member.

9) SAD NEWS (OUT OF BALANCE MOMENTS - second OOB$_{I}$)

$$\text{OOB}_{I} = \Sigma \text{COM}_{II} \text{ OF ALL MEMBERS ENTERING INTO JOINT I}$$

The sad news is that when these moments carry over to the other ends of the members, they cause those joints to again go out of balance. You can check this by simply adding up all the moments on the COM$_{II}$ (carry over moments) line of your sheet. Thus for each joint you will have to sum all the moments which came into that joint, after the carry overs, and compute a new OOB$_{I}$ (out of balance) moment.

10) DISTRIBUTE MOMENTS - second DIST$_{II}$

$$\text{DIST}_{II} = \text{OOB}_{I} \times \text{DF}_{II}$$

Now these out of balance moments must be distributed into the adjacent members. You compute the newly distributed moments by multiplying the out of balance moments (OOB$_{I}$) by the distribution factors (DF$_{II}$). Don't forget the minus sign!

11) CARRY OVER MOMENTS - second COM$_{II}$

$$\text{COM}_{II} = \text{DIST}_{II} \times \text{COF}_{II}$$

Now these newly distributed moments will carry over 1/2 of their values (or maybe none, depending on the carry over factor), to the other end of the member. Multiply the distributed moments by the carry over factors, and enter into the table at the far end of the member.

12) REALLY SAD NEWS - THIS NEVER ENDS!

Well, not really. When the worst out of balance moment around any joint get small enough that you just don't care anymore, quit. You should always quit after a carry over cycle, and then just ignore any out of balance you can live with.

The table below can be used for moment distribution:

Member Name - Give each member a unique name.
$L_{IJ}$ - The length of the member $IJ$, in any convenient units. You can even input $L$ as feet, and $E$ in ksi, since the error in units (12 inches per foot) will cancel.

$EI_{IJ}$ - The modulus of elasticity of member $IJ$.

$K_{IJ}$ - The computed relative stiffness for end I of member $IJ$.

$DF_{IJ}$ - The computed distribution factor for end I of member $IJ$.

$COF_{IJ}$ - The computed carry over factor for end I of member $IJ$.

$FEM_{IJ}$ - The computed fixed end moment for end I of member $IJ$.

$OOB_{I}$ - The computed out of balance moment at I, found by summing the FEM's around joint I.

$DIST_{IJ}$ - The computed distribution moment going to end I of member $IJ$, as a result of $OOB_{I}$.

$COM_{II}$ - The computed carry over moment going to the other end of member $IJ$.

The final answer for the moment at the end of each member is found by adding:

$$\text{MOMENT FINAL}_{IJ} = \text{FEM}_{IJ} + \sum \text{DIST}_{IJ} + \sum \text{COM}_{II}$$

i.e. simply add the FEM at the head of the column, plus all other moments under it.

DO NOT ADD IN ANY OF THE OUT OF BALANCE MOMENTS ($OOB_{I}$). These were merely written down as a convenience to compute the $DIST_{IJ}$ values.
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DRAWING SHEAR AND MOMENT DIAGRAMS FOR THE MEMBERS

The final resulting moments are obtained from the moment distribution sheet above. Remember that the FEM moments, and all those below it are added into the sum, but NOT the OOB moments, which are in a different column.

The member end moments are found as listed in the Final Moment column. These moments act clockwise on the end of the member if the resulting sum was positive, counter clockwise if negative. These end moments are simply put on the end of a free-body of the member, along with the transverse loads (concentrated and/or uniform), and any concentrated moments, and the moment diagram is drawn as before. Maximum shears and moments are then used for design of the member.