Professional Engineering Review Examination

Session 4: Concrete Analysis and Design

By:

Joseph M. Bracci, Ph.D., P.E.
Assistant Professor of Civil Engineering
Texas A&M University
Concrete Mixing

(a) Types of Concrete (Portland Cement, PC):

- **Type I**: Normal PC - General Purpose
- **Type II**: Modified PC - Moderate sulfate resistance, used in hot weather for large structures
- **Type III**: High Early Strength
- **Type IV**: Low Heat - for large structures
- **Type V**: Sulfate Resistant
(b) Proportioning Concrete

- Designated as a ratio of cement : fine aggr. : coarse aggr.  
  (ie, 1 : 2 : 3)

- Ratios can be in terms of weight or volume

- Amount of water is usually quantified in terms of gallons of water per 94 lb. sack of concrete (or water-cement ratio)

**In-Place Volume (Absolute Volume Method)**

Amount of Concrete = Sum of the solid volumes of cement, sand, coarse aggregates, and water.

**Typical Specific Weights:**

cement - 195 pcf; fine aggregates - 165 pcf; coarse aggregates - 165 pcf; water - 62.4 pcf
1 cement sack weighs 94 pounds.
7.48 gallons = 1 cf volume  (or 239.7 gal = 1 ton of water)

**Air Entrainment:** The yield is increased by additional air. Therefore divide the solid yield by (1 - air %).

**Water content in aggregate** above saturated surface dry (SSD) water content must be subtracted from the water required.

**Water content below the SSD** water content must be added to the water required.

Densities should be the SSD densities.
Example 14.1

A mix is designed as 1:1.9:2.8 by weight. The water-cement ratio is 7 gallons of water per sack of cement. (a) What is the concrete yield in cubic feet per sack of cement? (b) How much of each constituent is needed to make 45 cubic yards of concrete?

(a) The solution can be tabulated as follows:

<table>
<thead>
<tr>
<th>material</th>
<th>ratio</th>
<th>weight per sack</th>
<th>solid density</th>
<th>absolute volume (ft³/sack)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement</td>
<td>1.0</td>
<td>(1 \times 94 = 94)</td>
<td>195</td>
<td>(94/195 = 0.48)</td>
</tr>
<tr>
<td>sand</td>
<td>1.9</td>
<td>(1.9 \times 94 = 179)</td>
<td>165</td>
<td>(179/165 = 1.08)</td>
</tr>
<tr>
<td>coarse</td>
<td>2.8</td>
<td>(2.8 \times 94 = 263)</td>
<td>165</td>
<td>(263/165 = 1.60)</td>
</tr>
<tr>
<td>water</td>
<td></td>
<td></td>
<td></td>
<td>(7/7.48 = 0.94)</td>
</tr>
</tbody>
</table>

The yield is 4.1 cubic feet of concrete per sack cement.
(b) The number of one-sack batches is

\[
\frac{(45) \text{yd}^3(27) \text{ft}^3/\text{yd}^3}{(4.1) \text{ft}^3/\text{sack}} = 296.3 \text{sacks } (\text{say 297})^4
\]

Order 297 sacks of cement.

\[
\frac{(297)(1.9)(94)}{2000} = 26.5 \text{ tons of sand}
\]

\[
\frac{(297)(2.8)(94)}{2000} = 39.1 \text{ tons of coarse aggregate}
\]

\[
(297)(7) = 2079 \text{ gallons of water}
\]
Example 14.2

50 cubic feet of 1:2\frac{1}{2}:4 (by weight) concrete are to be produced. The constituents have the following properties:

<table>
<thead>
<tr>
<th>constituent</th>
<th>SSD density (pcf)</th>
<th>moisture (dry basis from SSD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement</td>
<td>197</td>
<td>-</td>
</tr>
<tr>
<td>sand</td>
<td>164</td>
<td>5% excess</td>
</tr>
<tr>
<td>coarse aggregate</td>
<td>168</td>
<td>2% deficit</td>
</tr>
</tbody>
</table>

What are the required order quantities if the design calls for 5.5 gallons of water per sack and 6% entrained air?

<table>
<thead>
<tr>
<th>weight per sack SSD absolute volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>constituent ratio cement density</td>
</tr>
<tr>
<td>cement 1.0 94 197 0.477</td>
</tr>
<tr>
<td>sand 2.5 235 164 1.433</td>
</tr>
<tr>
<td>coarse 4.0 376 168 2.238</td>
</tr>
<tr>
<td>water 5.5/7.48 = 0.735 4.883 ft³/sack</td>
</tr>
</tbody>
</table>

The yield with 6% air is

\[
\frac{4.883}{1 - 0.06} = 5.19 \text{ ft}^3/\text{sack}
\]
The number of one sack batches is

\[ \frac{50}{5.19} = 9.63 \]

(In practice, this would be rounded up.)

The required sand weight (ordered as is, not SSD) is

\[ \frac{(9.63)(1.05)(94)(2.5)}{2000} = 1.19 \text{ tons} \]

The required coarse aggregate weight (ordered as is, not SSD) is

\[ \frac{(9.63)(0.98)(94)(4)}{2000} = 1.77 \text{ tons} \]

The excess water contained in the sand is

\[ (1.19) \left( \frac{0.05}{1.05} \right) (239.7) \text{ gal/ton} = 13.58 \text{ gallons} \]

The water needed to bring the coarse aggregate to SSD conditions is

\[ (1.77) \left( \frac{0.02}{0.98} \right) (239.7) \text{ gal/ton} = 8.66 \text{ gallons} \]
The total water needed is

\[(5.5)(9.63) + 8.66 - 13.58 = 48.0 \text{ gallons}\]
Properties Of Concrete

(a) **Slump:** Measure of concrete plasticity
   Typically 1” - 4”

(b) **Compressive Strength** ($f'_c$):
    Cylinder tests
    \[ f'_c = 2000 \text{ psi} - 8000 \text{ psi} \text{ (normal concrete)} \]
    \[ f'_c = 8000 \text{ psi} - 25000 \text{ psi} \text{ (high strength concrete)} \]

(c) **Tensile Strength:**
    Typically (7% - 10%) $f'_c$
    Normally found from split or rupture tests
    \[ \text{Rupture modulus} = 7.5 \sqrt{f'_c} \] \text{ (} f'_c \text{ in psi)}
    \text{ (ACI for cracking strength)}
(d) Shear Strength:

Typically (16\% - 25\%) f'_c

\[ V_c = 2 \sqrt{f'_c} \cdot b \cdot d \] (ACI 11.3.1.1) \ (f'_c \text{ in psi})

(e) Density (specific weight):

Normal reinforced concrete \sim 150 \text{ pcf}

Lightweight concrete \sim 120 \text{ pcf}

(f) Modulus of Elasticity (secant stiffness):

\[ E_c = w^{1.5} \cdot (33) \cdot \sqrt{f'_c} \quad = \quad 57,000 \cdot \sqrt{f'_c} \] \ (f'_c \text{ in psi})
(g) Concrete Compressive Stress vs. Strain Behavior:

- Initial Stiffness
- Confined
- Unconfined
- Secant Stiffness
- $f_{cc}'$
- $0.85 f_{c}'$
- Strain
  - 0.002
  - 0.003

Stress Axis
Properties Of Steel (Rebar)

(a) Yield Strength --- Grade of Steel (40 ksi or 60 ksi)

Initial Stiffness = 29000 ksi

Stress

fy = 60 ksi

fy = 40 ksi

Strain
(b) ASTM Standard Rebar Properties

### ASTM Standard Reinforcing Bars

<table>
<thead>
<tr>
<th>size</th>
<th>weight (lb/ft)</th>
<th>diameter (in)</th>
<th>area (in²)</th>
<th>perimeter (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2</td>
<td>0.167</td>
<td>0.250</td>
<td>0.05</td>
<td>0.786</td>
</tr>
<tr>
<td>#3</td>
<td>0.376</td>
<td>0.375</td>
<td>0.11</td>
<td>1.178</td>
</tr>
<tr>
<td>#4</td>
<td>0.668</td>
<td>0.500</td>
<td>0.20</td>
<td>1.571</td>
</tr>
<tr>
<td>#5</td>
<td>1.043</td>
<td>0.625</td>
<td>0.31</td>
<td>1.963</td>
</tr>
<tr>
<td>#6</td>
<td>1.502</td>
<td>0.750</td>
<td>0.44</td>
<td>2.356</td>
</tr>
<tr>
<td>#7</td>
<td>2.044</td>
<td>0.875</td>
<td>0.60</td>
<td>2.749</td>
</tr>
<tr>
<td>#8</td>
<td>2.670</td>
<td>1.000</td>
<td>0.79</td>
<td>3.142</td>
</tr>
<tr>
<td>#9</td>
<td>3.400</td>
<td>1.128</td>
<td>1.00</td>
<td>3.544</td>
</tr>
<tr>
<td>#10</td>
<td>4.303</td>
<td>1.270</td>
<td>1.27</td>
<td>3.990</td>
</tr>
<tr>
<td>#11</td>
<td>5.313</td>
<td>1.410</td>
<td>1.56</td>
<td>4.430</td>
</tr>
<tr>
<td>#14</td>
<td>7.65</td>
<td>1.593</td>
<td>2.25</td>
<td>5.32</td>
</tr>
<tr>
<td>#18</td>
<td>13.60</td>
<td>2.257</td>
<td>4.00</td>
<td>7.09</td>
</tr>
</tbody>
</table>
Ultimate Strength Design

- Actual loads are multiplied by load factors, $\alpha$, and compared with loads that would cause failure!

- The ultimate strength is also multiplied by a capacity reduction factor, $\phi$, to account for workmanship and material understrength.

$$\phi M_n = \alpha_i M_D + \alpha_j M_L + \ldots$$
### Strength Reduction Factors (ACI 9.3)

<table>
<thead>
<tr>
<th>Type of Stress</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexure</td>
<td>0.90</td>
</tr>
<tr>
<td>Axial Tension</td>
<td>0.90</td>
</tr>
<tr>
<td>Shear/Torsion</td>
<td>0.85</td>
</tr>
<tr>
<td>Axial Compress w/ spirals</td>
<td>0.75</td>
</tr>
<tr>
<td>Axial Compress w/ ties</td>
<td>0.70</td>
</tr>
<tr>
<td>Bearing</td>
<td>0.70</td>
</tr>
<tr>
<td>Unreinforced</td>
<td>0.65</td>
</tr>
</tbody>
</table>

### Required Strengths (ACI 9.2)

(i) $U = 1.4 \, D + 1.7 \, L$
(ii) $U = 0.75 \, (1.4 \, D + 1.7 \, L + 1.7 \, W)$
(iii) $U = 0.9 \, D + 1.3 \, W$
(iv) $U = 1.05 \, D + 1.28 \, L + 1.40 \, E$
(v)  $U = 0.9 \, D + 1.43 \, E$
Beams in Bending

Assumptions:

(i) Plane sections remain plane (linear $\varepsilon$ diagram)

(ii) Perfect bond between rebar and concrete

(iii) $\sigma = E \varepsilon$ when the stresses are below cracking

(iv) Neglect concrete in tension beyond cracking

Loading Stages:

(i) Uncracked section up to cracking ($M_{\text{crack}}$)

(ii) Cracked section beyond cracking ($M_{\text{yield}}$)

(iii) Ultimate strength ($M_{\text{ult}}$)
Stage 1: Uncracked Section

\[ \varepsilon_s = \varepsilon_{cs} \]

\[ \frac{f_s}{E_s} = \frac{f_{cs}}{E_c} \implies f_s = n \cdot f_{cs} \]

\[ f_{cb} = M \cdot \frac{y_b}{I_g} \implies M_{cr} = f_r \cdot \frac{I_g}{y_b} \]
where \( f_r = 7.5 \sqrt{f_c'} \), Rupture Modulus (\( f_c' \) in psi)

\[ l_g = \text{gross moment of inertia, neglecting rebar} \]

\[ y_b = \text{distance from NA to extreme tension fiber (} h / 2) \]
Stage 2: Cracked Section

1. Equilibrium: \( C_c = T \)
   \[
   f_c \frac{b x}{2} = A_s f_s \quad \Rightarrow \quad f_s = f_c \frac{b x}{2 A_s}
   \]

2. Compatibility:
   \[
   \frac{\varepsilon_s}{d-x} = \varepsilon_c / x
   \]
3. Constitutive Law (Hooke's):

\[ f_s = E_s \varepsilon_s \quad & \quad f_c = E_c \varepsilon_c \]

4. Substitute 2 into 3 into 1:

\[ x^2 + 2 n A_s x / b - 2 n A_s d / b = 0 \]

5. Let \( \rho = A_s / (b d) \)

\[ k_{cr} = x / d \]

6. Therefore depth to neutral axis,

\[ k_{cr} = \sqrt{ (\rho n)^2 + (2\rho n) - (\rho n)} \]
7. Cracked moment of Inertia:

\[ x = k_{cr} \cdot d \]

\[ l_{cr} = b \cdot x^3 / 3 + n \cdot A_s \cdot (d - x)^2 \]

8. Yield Moment:

\[ M_y = A_s \cdot f_y \cdot j \cdot d = A_s \cdot f_y \cdot d \cdot [1 - (k_{cr} / 3)] \]
Stage 3: Ultimate Strength

* Use Whitney Stress Block Assumptions and Assume Steel Yields

1. Equilibrium: \( C_c = T = A_s \cdot f_y \)

2. \( \Rightarrow \quad a = A_s \cdot f_y / (0.85 \cdot f_c' \cdot b) \)

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3. or \[ k = A_s f_y / (0.85 f_c b d) \]

4. Ultimate Moment:

\[ M_{ult} = A_s f_y j d = A_s f_y d \left[ 1 - \left( \frac{k}{2} \right) \right] \]

*Typical Moment -vs- Curvature Response*
Analysis of Singly Reinforced Rectangular Beams

Given: b, d, As, $f'_c$, $f_y$
Take: $E_s = 29000$ ksi

Assume Rebar Yields $\implies f_s = f_y$

Calculate $k = \frac{As \cdot f_y}{(0.85 \cdot f'_c \cdot b \cdot d)}$

Check $k \min = \frac{200}{(0.85 \cdot f'_c)}$
\[
\beta_1 = 0.85 - 0.05 \left( f'_c - 4000 \right) / 1000 \quad \text{\{4000} \leq f'_c \leq 8000\) \text{ \{0.65} \leq \beta_1 \leq 0.85\}
\]

Calculate \( k_{bal} = \beta_1 \left( 87000 \right) / (87000 + f_y) \)

\[
M_n = A_s f_y d \left( 1 - k / 2 \right)
\]
Design of Singly Reinforced Rectangular Beams

(i). \[ \phi M_n \geq M_{\text{demand}} \ (\text{i.e.,} \ 1.4 \ M_{DL} + 1.7 \ M_{LL}), \quad \phi_{\text{bending}} = 0.90 \]

(ii). \[ k \leq 0.75 \ k_{bal} \] This ensures a ductile failure.

(iii). Select \( h \) based on deflection requirement from Table 9.5a (ACI)

(iv). Choose \( f'c, \ f_y, \ b, \) and \( A_s \)

(v). Choose \( b : h \sim (1 : 1.5 \ \text{to} \ 1 : 2) \)
## Minimum Beam Width and Number of Bars per Layer

<table>
<thead>
<tr>
<th>size of bars</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>add for each added bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>#4</td>
<td>6.1</td>
<td>7.6</td>
<td>9.1</td>
<td>10.6</td>
<td>12.1</td>
<td>13.6</td>
<td>15.1</td>
<td>1.50</td>
</tr>
<tr>
<td>#5</td>
<td>6.3</td>
<td>7.9</td>
<td>9.6</td>
<td>11.2</td>
<td>12.8</td>
<td>14.4</td>
<td>16.1</td>
<td>1.63</td>
</tr>
<tr>
<td>#6</td>
<td>6.5</td>
<td>8.3</td>
<td>10.0</td>
<td>11.8</td>
<td>13.5</td>
<td>15.3</td>
<td>17.0</td>
<td>1.75</td>
</tr>
<tr>
<td>#7</td>
<td>6.7</td>
<td>8.6</td>
<td>10.5</td>
<td>12.4</td>
<td>14.2</td>
<td>16.1</td>
<td>18.0</td>
<td>1.88</td>
</tr>
<tr>
<td>#8</td>
<td>6.9</td>
<td>8.9</td>
<td>10.9</td>
<td>12.9</td>
<td>14.9</td>
<td>16.9</td>
<td>18.9</td>
<td>2.00</td>
</tr>
<tr>
<td>#9</td>
<td>7.3</td>
<td>9.5</td>
<td>11.8</td>
<td>14.0</td>
<td>16.3</td>
<td>18.6</td>
<td>20.8</td>
<td>2.26</td>
</tr>
<tr>
<td>#10</td>
<td>7.7</td>
<td>10.2</td>
<td>12.3</td>
<td>15.3</td>
<td>17.8</td>
<td>20.4</td>
<td>22.9</td>
<td>2.54</td>
</tr>
<tr>
<td>#11</td>
<td>8.0</td>
<td>10.8</td>
<td>13.7</td>
<td>16.5</td>
<td>19.3</td>
<td>22.1</td>
<td>24.9</td>
<td>2.82</td>
</tr>
<tr>
<td>#14</td>
<td>8.9</td>
<td>12.3</td>
<td>15.6</td>
<td>19.0</td>
<td>22.4</td>
<td>25.8</td>
<td>29.2</td>
<td>3.39</td>
</tr>
<tr>
<td>#18</td>
<td>10.5</td>
<td>15.0</td>
<td>19.5</td>
<td>24.0</td>
<td>28.6</td>
<td>33.1</td>
<td>37.6</td>
<td>4.51</td>
</tr>
</tbody>
</table>
Design Process: (b, d, and As are unknowns)

1. Determine Demand Moment, $M_{\text{demand}}$ (i.e. 1.4 $M_{\text{DL}}$ + 1.7 $M_{\text{LL}}$)

2. Select $f_c'$ and $f_y$

3. Select $k = 0.30$ $k_{\text{bal}}$ (recall $k_{\text{max}} = 0.75$ $k_{\text{bal}}$)

   \[
   \{ k_{\text{bal}} = \frac{\beta_1 (87000)}{(87000 + f_y)} \}
   \]

4. Assume $b$ and find $d$ ($b/h \sim 1/1.5$ to $1/2.0$, if possible)

   \[ bd^2 = \frac{(M_{\text{demand}} / \phi)}{[0.85 f_c' k (1 - k/2)]} \]

5. Find $A_s$, req'd = \[ (M_{\text{demand}} / \phi) / [f_y d (1 - k/2)] \]

6. Check $A_s$, min = 200 b d / $f_y$

7. Select Reinforcement from table
8. Find \( k_{\text{prov}} = \rho_{\text{prov}} \, f_y / (0.85 \, f_{c'}) \)

9. Determine \( M_{\text{prov}} = \phi \, A_s \, f_y \, d \left( 1 - k_{\text{prov}} / 2 \right) \)

10. Determine \( h = d + \text{cover} + d_{b, \text{stirrup}} + 1/2 \, d_{b, \text{long}} \)

11. Determine actual \( d \) ---\( \rightarrow \) actual \( k \) ---\( \rightarrow \) actual moment

**Design Process:** (\( h \) is restricted or known, and \( b \) and \( A_s \) are unknowns)

1. Determine Demand Moment, \( M_{\text{demand}} \) (ie. 1.4 \( M_d \) + 1.7 \( M_l \))

2. Select \( f_{c'} \) and \( f_y \)

3. Assume \( b \) and compute \( d \) based on known \( h \)

4. Calculate \( k = 1 - \sqrt{1 - (2 \, M_{\text{demand}} / \phi) / 0.85 \, f_{c'} \, b \, d^2)} \)
5. Check if \( k \leq 0.75 k_{bal} \)

6. Find \( As, \text{req'd} = \frac{M_{demand}}{\phi} / [fy \cdot d \cdot (1 - k/2)] \)

7. Check \( As, \text{min} = 200 b \cdot d / fy \)

8. Select Reinforcement from table

9. Find \( k_{prov} = \rho_{prov} \cdot fy / (0.85 fc') \)

10. Determine \( M_{prov} = \phi As \cdot fy \cdot d \cdot (1 - k_{prov} /2) \)
Ex. Design a rectangular beam with tension reinforcement to carry service moments of 34,300 ft-lb (dead) and 30,000 ft-lb (live). Use fc' = 3500 psi, fy = 40,000 psi, and #3 bars for shear.

- \( M_{\text{demand}} = 1.4 \, M_{\text{DL}} + 1.7 \, M_{\text{LL}} = 1.4(34.3) + 1.7(30) = 99 \, \text{kip-ft} \)

- Select \( k = 0.30 \, k_{\text{bal}} = 0.30 \{87/ (87 + 40)\} = 0.206 \)

- \( bd^2 = (M_{\text{demand}} / \phi) / \{ 0.85 \, fc' \, k (1 - k / 2)\} \)
  
  \[ = (99.2 \, (12) / 0.9) / \{ 0.85 \, (3.5) \, (0.206) \, (1 - 0.206 / 2)\} \]
  
  \[ = 2406 \, \text{in}^3 \]

- use \( b = 9 \, \text{in} \), therefore \( d = 16.35 \, \text{in} \)

- \( As, \text{req'd} = (M_{\text{demand}} / \phi) / \{ fy \, d \, (1 - k / 2)\} \)
  
  \[ = (99 \, (12) / 0.9) / \{ 40 \, (16.35) \, (1 - 0.206 / 2)\} = 2.25 \, \text{in}^2 \]

- \( As, \text{min} = 200 \, b \, d / fy = 200 \, (9) \, (16.35) / 40000 = 0.74 \, \text{in}^2 \quad \text{OK}!!! \)
- Use 3 #8 bars (As, prov = 2.37 in\(^2\))

- \( h = d + \text{dia } #8 / 2 + \text{dia } #3 \text{ stirrup} + \text{cover} \)
  \[ = 16.35 + 0.5 + 0.375 + 1.5 = 18.725 \text{ in} \quad \Rightarrow \quad \text{use 19 in} \]

- \( d_{act} = 19 - 1.5 - 0.375 - 0.5 = 16.625 \text{ in} \)

- \( k_{prov} = \{ \frac{\text{As, prov}}{\text{bd}} \} \frac{f_y}{(0.85 \, f_{c'}')} \)
  \[ = \{ \frac{2.37}{(9 \times 16.625)} \} \frac{40}{(0.85 \times 3.5)} = 0.213 \quad (\leq 0.75 \, k_{bal}) \]

- \( M_{act} = \phi(\text{As, prov}) f_y d \left( 1 - \frac{k_{prov}}{2} \right) = 1267.4 \text{ k-in} = 105.6 \text{ k-ft} \)
  \[ \geq 99 \text{ k-ft} \quad \text{OK !!!} \]
Design and Analysis of T/L Sections

- Effective Flange Width (ACI 8.10.2):

![Diagram of T/L Section with dimensions](image)

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>$B_T$</th>
<th>$B_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Built-In (Monolithic)</td>
<td>$\leq \frac{L}{4}$, $\leq b + 16t$, $\leq So$</td>
<td>$\leq b + \frac{L}{12}$, $\leq b + 6t$, $\leq b + \frac{So}{2}$</td>
</tr>
<tr>
<td>Isolated</td>
<td>$B_T \leq 4b$ &amp; $b \leq 2t$</td>
<td></td>
</tr>
</tbody>
</table>
- Analysis of T-Sections:

\[ Mn = 0.85 \cdot f_c' \cdot (B-b) \cdot t \cdot (d - t/2) + 0.85 \cdot f_c' \cdot b \cdot kd \cdot (d - kd/2) \]

\[ As, fl = \frac{Cf}{fy} = 0.85 \cdot f_c' \cdot (B-b) \cdot t / fy \]

\[ As, web = 0.85 \cdot f_c' \cdot b \cdot kd / fy \]

\[ As, tot = As, fl + As, web \]

Only Applies when \( kd > t \) !!!

When \( kd \leq t \), singly reinforced with width B
**T-Section Design Procedure**  (Given $M_{demand}$ & $kd > t$)

1. Find $M, fl = 0.85 fc'(B-b) t (d-t/2)$

2. Find required $M, web = \left( \frac{M_{demand}}{\phi} \right) - M, fl$

3. Compute size of compression block:

   $$k = 1 - \sqrt{1 - \frac{(2 M, web)(0.85 fc' b d^2)}{2 M, web}}$$

4. Find $As, fl = \frac{M, fl}{(arm \cdot fy)} = 0.85 fc'(B-b) t / fy$

5. Find $As, web = \frac{M, web}{(arm \cdot fy)} = \frac{M, web}{fy \cdot (d - kd/2)}$

6. Total Reinforcement $As, tot = As, fl + As, web$

7. Check $As, tot \geq As, min = 200 b d / fy$
8. Check \( \rho \leq 0.75 \rho_{\text{bal}} \)

- \( k_{\text{bal}} = \beta_1 \left[ 87000 \div (87000 + f_y) \right] \)

- \( C_{\text{bal}} = C_w + C_w \)
  
  \[ = 0.85 f'_c (B - b) (t) + 0.85 f'_c (b) (k_{\text{bal}}) (d) \]

- \( A_{s, \text{bal}} = C_{\text{bal}} / f_y \)

- Is \( A_{s, \text{tot}} \leq 0.75 A_{s, \text{bal}} \)?
Doubly Reinforced Sections

Design of Doubly Reinforced Beams in Bending

(Given $M_{\text{demand}}$, b, d, $f_c'$, and fy)

1. Determine maximum moment that can be taken by singly reinforced beam.
- Determine desired $k_{max} \ (\ = 0.75 \ k_{bal})$

- Determine moment capacity, $M_1$
  
  $M_1 = 0.85 \ fc' \ b \ d^2 \ k_{max} \ (1 - k_{max} / 2)$

- Determine $As_1$, req to fit $M_1$
  
  $As_1, \ req = M_1 / [fy \ (1 - k_{max} / 2) \ d]$

2. Determine if doubly reinforced section is req'd
   
   Is $M_{demand} / \phi \ > \ M_1$ ?
   
   if yes, then $M_2 = M_{demand} / \phi - M_1$

3. Negative Steel :
   
   $As', \ req'd = M_2 / [(d-d') \ * \ (fy - 0.85 \ fc')]$

4. Postive Steel (Assumes negative steel yields) :
   
   $As, \ req'd = As', \ req'd + As_1, \ req'd$
5. Does negative steel yield?

\[ \varepsilon_s' = \varepsilon_{cu} \left( k - \beta \frac{d'}{d} \right) / k \]

if \( \varepsilon_s' \geq \varepsilon_y \), then use \( f_y \)

else use \( f_s = E_s \varepsilon_s' \) above
ACI Moment Coefficients (ACI 8.3.3)

(a) Terminology

(b) Moment coefficients—Discontinuous end unrestrained, more than 2 spans

(c) Moment coefficients—Discontinuous end integral with support where support is a spandrel girder

(d) Moment coefficients—Discontinuous end integral with support where support is a column
Development Length

Basic Development Length (Tension)
#11 Bars and smaller (ACI 12.2.2)
\[ L_{\text{db}} = 0.04 \ A_b \ \frac{f_y}{\sqrt{f_{c'}}} \ \text{or} \ 12'' \]

Factors:
- spacing and cover (ACI 12.2.3.1)
  Satisfactory \( \Rightarrow \) factor = 1.0

- (ACI 12.3.2) cover = \( d_b \) or less with
  clear spacing 2 \( d_b \) or less
  \( \Rightarrow \) factor = 2.0

- If neither category satisfied (ACI 12.2.3.3)
  factor = 1.4

- clear spacing \( \geq 5 \ d_b \) and cover \( \geq 2.5 \ d_b \) (ACI 12.2.3.4)
  factor = 0.8
- Top Reinforcement w/ 12” concrete under bar (ACI 12.2.4.1)
  factor = 1.3

- Excess Reinforcement (ACI 12.2.5)
  factor = As, req / As, prov

- Epoxy coated rebar (ACI 12.2.4.3)
- Lightweight concrete (ACI 12.2.4.2)

**Basic Compression Development Length (ACI 12.3)**

- \( L_{dbc} = 0.02 \, d_b \, f_y \, / \sqrt{f_{c'}^{'}} \geq 0.0003 \, d_b \, f_y \)

- Factors apply for spirals (ACI 12.2.3.5) and excess reinforcement (ACI 12.2.5)

- \( L_{dc} \geq 8” \)
Hooked Anchorages (Detail ACI 7.1)

\[ \text{L}_{dh} = \text{L}_{hb} \times \text{Factors} \]

\[ \text{L}_{hb} = \frac{1200 \ d_b}{\sqrt{\text{fc}'}} \quad \text{for } f_y = 60,000 \text{ psi} \]

\[ \text{L}_{dh} \geq 8 \ d_b \quad \text{and} \quad \text{L}_{dh} \geq 6" \]

**Factors:**
- (ACI 12.5.3.1) grade steel
  \[ \text{factor} = \frac{f_y}{60,000} \]

- (ACI 12.5.3.2a) 180 degree hooks and 2.5” side cover
  \[ \text{factor} = 0.7 \]

- (ACI 12.5.3.2b) 90 degree hooks, 2.5” side cover, and tail \( \geq 2" \)
  \[ \text{factor} = 0.7 \]
-(ACI 12.5.3.3) confinement
  factor = 0.8

- (ACI 12.5.3.4) excess reinforcement
  factor = As, req / As, prov

-(ACI 12.5.3.5) lightweight concrete
  factor = 1.3

**Cutoffs**

(ACI 12.10.3) - All bars must cover design envelope and provide an extra 12 \( d_b \) or \( d \) (larger), plus sufficient development length for bars.
## Tension Lap Splices (ACI 12.15)

<table>
<thead>
<tr>
<th>As,req/As,prov</th>
<th>%As Spliced</th>
<th>Splice Class</th>
<th>Lap, req’d</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 0.5</td>
<td>≤ 75</td>
<td>A</td>
<td>$L_d$</td>
<td>Desirable</td>
</tr>
<tr>
<td></td>
<td>&gt; 75</td>
<td>B</td>
<td>1.3 $L_d$</td>
<td>ck</td>
</tr>
<tr>
<td>&gt; 0.5</td>
<td>≤ 50</td>
<td>B</td>
<td>1.3 $L_d$</td>
<td>ck</td>
</tr>
<tr>
<td></td>
<td>&gt; 50</td>
<td>C</td>
<td>1.7 $L_d$</td>
<td>Avoid</td>
</tr>
</tbody>
</table>

where $As, \text{req'd} = \text{determined for bending}$  
$L_d = \text{development length for bars} \geq 12”$

## Compression Lap Splices (ACI 12.16)

\[
\begin{align*}
\text{Lap, req’d} &= 0.0005 \, fy \, d_b \\
&\quad \text{for } fy \leq 60,000 \, \text{psi} \\
\text{Lap, req’d} &= (0.0009 \, fy - 24) \, d_b \\
&\quad \text{for } fy > 60,000 \, \text{psi} \\
\text{Lap, req’d} &\geq 12”
\end{align*}
\]
Lap Splices in Columns (bars may be in tension or compression)  
(ACI 12.17.2.2)

<table>
<thead>
<tr>
<th>Tension Bar Stress</th>
<th>% Splice</th>
<th>Splice Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_s^+ \leq \frac{fy}{2}$</td>
<td>$&gt; 50$</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>$\leq 50$</td>
<td>A</td>
</tr>
<tr>
<td>$f_s^+ &gt; \frac{fy}{2}$</td>
<td></td>
<td>B</td>
</tr>
</tbody>
</table>

In tied columns with ties $\geq 0.0015$ hs, factor $= 0.83$
In spiral columns, factor $= 0.75$

But Splice length $\geq 12$’
Shear Resistance of R/C Members

Total Shear Resistance = \( V_n = V_c + V_s \)

where \( V_c = 2 \sqrt{f_{c'}} b d \) -beams
\[
V_c = 2 \left( 1 + \frac{N}{[2000 \text{ Ag}]} \right) \sqrt{f_{c'}} b d
\] -columns in compress.
\[
V_c = 2 \left( 1 + \frac{N}{[500 \text{ Ag}]} \right) \sqrt{f_{c'}} b d
\] -columns in tension

\( N \) = axial compressive load
\( b \) = member width
\( d \) = member depth to long. rebar
\( \text{Ag} \) = member area

\( V_s = Av f_y d / s \)

\( Av \) = total stirrup area
\( s \) = stirrup spacing
Shear Design (ACI Section 11)

\[ V_{\text{demand}} \leq \phi_s \ V_n = \phi_s (V_c + V_s) \]

\[ \phi_s = 0.85 \text{ for shear} \]

Beams with Web Shear Reinforcement

(a) If \( V_{\text{demand}} \leq \phi_s V_c / 2 \), Then no shear rebar req'd

(b) If \( \phi_s V_c / 2 \leq V_{\text{demand}} \leq \phi_s V_c \), Then min rebar req'd

\[ - \text{Av, min} = 50 \text{ b s} / \text{fy} \quad \text{or} \quad \text{Vs, min} = 50 \text{ b d} \]
\[ - s \leq d / 2 \leq 24" \]

(c) If \( \phi_s V_c \leq V_{\text{demand}} \leq \phi_s (V_c + V_s, \text{min}) \), Then min req'd

\[ - \text{Av} = 50 \text{ b s} / \text{fy} \quad \text{or} \quad \text{Vs} = 50 \text{ b d} \]
\[ - s \leq d / 2 \leq 24" \]
(d) If \( \phi_s (V_c + V_s, \min) \leq V_{\text{demand}} \leq \phi_s (V_c + 4 \sqrt{f_c' b d}) \), Then

- \( V_s, \text{req'd} = V_{\text{demand}} / \phi_s - V_c \)
- \( A_s, \text{req'd} = (V_s, \text{req'd})(s) / (fy d) \)
- \( s, \text{max} \leq d/2 \leq 24" \)

(e) If \( \phi_s (V_c + 4 \sqrt{f_c' b d}) \leq V_{\text{demand}} \leq \phi_s (V_c + 8 \sqrt{f_c' b d}) \), Then

- \( V_s, \text{req'd} = V_{\text{demand}} / \phi_s - V_c \)
- \( A_s, \text{req'd} = (V_s, \text{req'd})(s) / (fy d) \)
- \( s, \text{max} \leq d/4 \leq 12" \)
If inclined stirrups are used (\( \delta = \) angle from horizontal)

\[
V_s = A_s f_y \left( \sin \delta + \cos \delta \right) \frac{d}{s}
\]

**NOTE:** Critical Section is a distance \( d \) from the column face.
Stirrup Anchorage Requirements (ACI 12.13)

(1) Each bend must enclose long. bar

(2) For #5 stirrups and smaller, use standard hooks 90°, 135°, 180° for any grade steel

(3) For #6 - #8 stirrups with fy = 40,000 psi, (2) applies

(4) For #6 - #8 stirrups with fy > 40,000 psi, requires a standard hook plus embedment between midheight and outside of hook at least \(0.014 \frac{d_b \cdot f_y}{\sqrt{f_{c'}}}\)
(5) (ACI 12.13.2.3) for welded wire fabric

(6) Lap splice stirrups may be used in deep beams

\[ \uparrow \downarrow > 1.3 \text{ ld} \]

(7) (ACI 7.11) requires closed stirrups in beams with compression steel, beams subject to reversals, and torsion.

(ACI 7.13.2.2) requires closed stirrups in all perimeter beams
### Minimum Thicknesses for Deflection Control (ACI Table 9.5a)

<table>
<thead>
<tr>
<th>Member</th>
<th>Simply Supported</th>
<th>One End Continuous</th>
<th>Both Ends Continuous</th>
<th>Cantilever</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-way Slab</td>
<td>L/20</td>
<td>L/24</td>
<td>L/28</td>
<td>L/10</td>
</tr>
<tr>
<td>Beam/Rib Slab</td>
<td>L/16</td>
<td>L/18.5</td>
<td>L/21</td>
<td>L/8</td>
</tr>
</tbody>
</table>

### Design of One-Way Slabs

- Design as 12” wide beams and find $A_s$, req’d in units of $\text{in}^2 / \text{ft}$

- Maximum bar spacing for bending $\leq 3 \, t_{\text{slab}}$ or 18”

- Check crack control (ACI 10.6.4)
- Shrinkage rebar due to thinness (cracks perpendicular to span)

\[
\text{As, min (shk/tmp) } = 0.0020 \ (12") \ (t_{\text{slab}}) \quad \text{for } f_y < 60 \text{ ksi}
\]

\[
\text{As, min (shk/tmp) } = 0.0018 \ (12") \ (t_{\text{slab}})(60/f_y) \quad \text{for } f_y \geq 60 \text{ ksi}
\]

Shrinkage steel spacing \( \leq 5 \ t_{\text{slab}} \) or 18”

- Minimum flexural steel is as shrinkage steel !!

Areas of Bars / foot width of Slab

Table 14.8
Average Steel Area per Foot of Width

<table>
<thead>
<tr>
<th>bar size number</th>
<th>nominal diameter (in.)</th>
<th>2</th>
<th>2(\frac{1}{2})</th>
<th>3</th>
<th>3(\frac{1}{2})</th>
<th>4</th>
<th>4(\frac{1}{2})</th>
<th>5</th>
<th>5(\frac{1}{2})</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.375</td>
<td>0.66</td>
<td>0.53</td>
<td>0.44</td>
<td>0.38</td>
<td>0.33</td>
<td>0.29</td>
<td>0.26</td>
<td>0.24</td>
<td>0.22</td>
<td>0.19</td>
<td>0.17</td>
<td>0.15</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.500</td>
<td>1.13</td>
<td>0.94</td>
<td>0.78</td>
<td>0.67</td>
<td>0.59</td>
<td>0.52</td>
<td>0.47</td>
<td>0.43</td>
<td>0.39</td>
<td>0.34</td>
<td>0.29</td>
<td>0.26</td>
<td>0.24</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>0.625</td>
<td>1.84</td>
<td>1.47</td>
<td>1.23</td>
<td>1.05</td>
<td>0.92</td>
<td>0.82</td>
<td>0.74</td>
<td>0.67</td>
<td>0.61</td>
<td>0.53</td>
<td>0.46</td>
<td>0.41</td>
<td>0.37</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>0.750</td>
<td>2.65</td>
<td>2.12</td>
<td>1.77</td>
<td>1.51</td>
<td>1.32</td>
<td>1.18</td>
<td>1.06</td>
<td>0.96</td>
<td>0.88</td>
<td>0.76</td>
<td>0.66</td>
<td>0.59</td>
<td>0.53</td>
<td>0.44</td>
</tr>
<tr>
<td>7</td>
<td>0.875</td>
<td>3.61</td>
<td>2.83</td>
<td>2.40</td>
<td>2.06</td>
<td>1.80</td>
<td>1.60</td>
<td>1.44</td>
<td>1.31</td>
<td>1.20</td>
<td>1.03</td>
<td>0.80</td>
<td>0.72</td>
<td>0.60</td>
<td>0.55</td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>3.77</td>
<td>3.14</td>
<td>2.69</td>
<td>2.36</td>
<td>2.09</td>
<td>1.88</td>
<td>1.71</td>
<td>1.57</td>
<td>1.35</td>
<td>1.18</td>
<td>1.05</td>
<td>0.94</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>9</td>
<td>1.128</td>
<td>4.80</td>
<td>4.00</td>
<td>3.43</td>
<td>3.00</td>
<td>2.67</td>
<td>2.40</td>
<td>2.18</td>
<td>2.00</td>
<td>1.71</td>
<td>1.50</td>
<td>1.33</td>
<td>1.20</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>10</td>
<td>1.270</td>
<td>5.06</td>
<td>4.34</td>
<td>3.80</td>
<td>3.37</td>
<td>3.04</td>
<td>2.76</td>
<td>2.53</td>
<td>2.31</td>
<td>2.07</td>
<td>1.89</td>
<td>1.69</td>
<td>1.52</td>
<td>1.27</td>
<td>1.27</td>
</tr>
<tr>
<td>11</td>
<td>1.410</td>
<td>6.25</td>
<td>5.36</td>
<td>4.69</td>
<td>4.17</td>
<td>3.75</td>
<td>3.41</td>
<td>3.12</td>
<td>2.68</td>
<td>2.34</td>
<td>2.08</td>
<td>1.87</td>
<td>1.56</td>
<td>1.32</td>
<td>1.27</td>
</tr>
</tbody>
</table>
Limits on Crack Width (ACI 10.6.4)

\[
z = f_s \sqrt[3]{d_c A}
\]

where \( f_s \) = service load stress on rebar (use 0.6 \( f_y \))
\( d_c \) = distance from extreme tension fiber to the center of tension rebar
\( A \) = effective tension area of concrete surrounding tension rebar divided by the number of tension bars (2 \( d_c b / \# \) bars)

If \( z > 175 \) for interior exposure or \( z > 145 \) for exterior exposure, then beam should be redesigned.
Beam Deflections

If ACI Table 9.5a is used to find thickness, deflections do not need calculating,
Else use deflections from elastic superposition and an equivalent moment of inertia (\( I_{eq} \)) for the member from ACI Eq. 9-7.

\[
I_{eq} = (\frac{M_{cr}}{M_{max}})^3 \cdot I_g + [1 - (\frac{M_{cr}}{M_{max}})^3] \cdot I_{cr}
\]

where \( M_{cr} = 7.5\sqrt{f_{c'}} \cdot I_g / y_b \)
\( M_{max} = \) Maximum unfactored service moment
\( I_g = \) gross moment of inertia (rectangle \( \rightarrow \) \( b \cdot h^3 / 12 \))
\( y_b = \) distance from NA to extreme tension fiber (\( h / 2 \))
\( I_{cr} = \) cracked moment of inertia

Long term deflections = \( \lambda \) \* Instantaneous deflections

where \( \lambda = \) (time factor) / (1 + 50 As' / bd)

- time factor = 2 for 5 yrs or more, etc (ACI 9.5.2.5)
2. A R/C beam supporting a roof is needed to span 27 feet (simply supported). It carries a dead load of 1 kip/ft (including beam wt) and a live load of 2 kips/ft. Assume $f_c' = 4$ ksi, $f_y = 60$ ksi, and $E_s = 36000$ ksi. Use ACI code with max. steel percentages permitted. (a) Use #11 bars and design a rectangular beam with $b = 14''$. Clear cover on the steel must be at least 1.5''. Do not design for shear. (b) Will the estimated crack sizes be within the requirements of the code if the member has interior exposure? (c) Calculate the instantaneous and long-term deflections. Assume 30% of live load is to be sustained.

$$- M_{\text{demand}} = 1.4 M_{DL} + 1.7 M_{LL} = (1.4*1 + 1.7*2) (27^2) / 8 = 437.4 \text{ k-ft}$$

$$- k = 0.75 k_{bal} = 0.75 (0.85) (87 / (87 + 60)) = 0.377$$

$$- bd^2 = (M_{\text{demand}} / \phi) / [0.85 f_c' k (1 - k / 2)]$$
$$= [437.4 (12) / 0.9] / [0.85 (4) (0.377) (1 - 0.377/2)]$$
$$= 5606.7 \text{ in}^3$$

$$- b = 14 \text{ in} \implies d = 20 \text{ in}$$
- As, req’d = \( \left( \frac{M_{\text{demand}}}{\phi} \right) / \left[ f_y \, d \, (1 - k/2) \right] \)
  = \[ \frac{437.4 \, (12)}{0.9} \] / \[(60)(20)(1 - 0.377/2)]
  = 6.0 \text{ in}^2 \quad \text{(maximum allowed)}

- Try 4 #11 bars \quad As, prov = 4 \times (1.56) = 6.24 \text{ in}^2

- Increase depth to 21” to account for additional steel area
  \( k = \left( \frac{As, prov}{bd} \right) \left( \frac{f_y}{0.85 \, f_{c'}} \right) \)
  = \[ \frac{6.24}{(14 \times 21)} \] \(\left( \frac{60}{0.85 \times 4} \right) = 0.375 < 0.377 \quad \text{OK!!} \)

- \( h = d + \text{cover} + \text{stirrup} + d_b/2 = 21 + 1.5 + 0.5 + 11/16 = 23.7 \text{ in} \)
  use \( h = 24” \quad \Rightarrow \quad d_{\text{act}} = 21.3” \)

\textbf{Crack Check:}

- \( f_s = 0.6 \, f_y = 36 \text{ ksi} \)
- \( d_c = 24 - 21.3 = 2.7”; \quad A = 2 \, d_c \, b / \# \text{bars} = 18.9 \text{ in}^2 \)
- \( z = f_s \sqrt[3]{d_c \, A} = 133.5 \text{ k/in} < 175 \text{ k/in} \quad \text{OK!!} \)
Deflections

- \( l_g = \frac{bh^3}{12} = 14 \times 24^3 / 12 = 16,128 \text{ in}^4 \)

- \( M_{\text{max}} = \frac{wl^2}{8} = (1 + 2) \frac{(27)^2}{8} = 273.4 \text{ k-ft (unfactored)} \)

- \( f_r = 7.5 \sqrt{fc'} = 7.5 \sqrt{4000} = 474.3 \text{ psi} \)

- \( y_b = \frac{h}{2} = 12'' \)

- \( M_{cr} = \frac{f_r l_g}{y_b} = \left[ 474.3 \times 16128 / 12 \right] / 12000 = 53.1 \text{ k-ft} \)

- \( \left( \frac{M_{cr}}{M_{\text{max}}} \right)^3 = \left( \frac{53.1}{273.4} \right)^3 = 0.0073 \)

- \( n = \frac{29,000,000}{57000\sqrt{4000}} = 8.04; \quad \rho = \frac{6.24}{(14 \times 21.3)} = 0.0209 \)
  \( \rho n = 0.1682 \)

- \( k_{cr} = \sqrt{\rho n^2 + 2\rho n} - \rho n = 0.4357 \)
- $x_{cr} = k_{cr} d = (0.4357) (21.3) = 9.28''$

- $I_{cr} = b x_{cr}^3 / 3 + n As (d - x_{cr})^2 = 10,978 \text{ in}^4$

- $I_{eq} = (M_{cr} / M_{max})^3 I_g + [1 - (M_{cr} / M_{max})^3] I_{cr} = 11,015.6 \text{ in}^4$

- $\delta_{inst} = 5 w L^4 / 384 E I$
  
  $= 5 (1 + 2) (1000/12) (27*12)^4 / [384*57000*\sqrt{4000} * 11015.6]$
  
  $= 0.903'' \text{ (with 100\% live load)}$

- Long term deflection with 30 \% live load

  \[ [1 + (0.3)*2] / 3 = 53 \% \text{ of the } w \text{ in the previous calculation} \]

- $\delta = 0.53 * 0.903 = 0.479''$

- $\rho' = 0 \implies \lambda = 2$

- $\delta_{long\ term} = 2 (0.479) + 0.903 = 1.86'' \text{ (with 30\% live load)}$
Columns

Types: Tied and spiral
Loads: Contain both axial load and bending moments

Axially Loaded Columns (No moment):

\[ P_n \geq P_{\text{demand}} \]

\[ P_n = 0.85 \cdot f_c' \cdot (A_g - A_s) + A_s \cdot f_y \]

\[ P_{\text{demand}} = 1.4 \cdot P_{DL} + 1.7 \cdot P_{LL} \] (for gravity load only)

Column Notes

1. Minimum width or diameter \( \geq 8" - 10"\)

2. Minimum reinforcement (ACI 10.9.1)
   - Reinforcement ratio = \( A_s / A_g \geq 1\% \)
   - Minimum of 4 bars required
(3) Maximum reinforcement (ACI 10.9.1)
   \[ \text{As} / \text{Ag} \ < \ 8\% \]
   - keep this ratio at about 3% max to avoid congestion

(4) Distance between long. bars \( \geq 1.5 \, d_b \)
   \[ \geq 1.5 \, " \]

(5) Cover \( \geq 1.5" \) interior exposure
    \[ \geq 2" \] exterior exposure

(6) Strength reduction factor \( \phi = 0.70 \) (ties)
    \[ = 0.75 \] (spirals)
Axial Load and Bending Moment Interaction Diagrams

\[ \frac{\sigma P_n}{A_g}, \text{ksi} \]

\[ \frac{\sigma P_n \times \frac{e}{h}}{A_g h} = \frac{\sigma M_n}{A_v h}, \text{ksi} \]
\[ \frac{\sigma P_n}{A_g} \times e = \frac{\sigma M_n}{A_g h}, \text{ ksi} \]
\[ \frac{\sigma_{P_n}}{A_g}, \text{ksi} \]

\[ \frac{\sigma_{P_n}}{A_g} \times \frac{e}{h} = \frac{\sigma M_n}{A_q h}, \text{ksi} \]
\[ \frac{\sigma_{P_n}}{A_g} \times \frac{e}{h} = \frac{\sigma_{M_n}}{A_y h}, \text{ksi} \]
Column Lap Splice Requirements

Axial Load vs. Moment

Compress. Lap Spl.:
- $fs = 0$
- Class A Tens. $fs = 0.5fy$
- Class B Tens.

Ties Requirements

1. Anti-buckling (ACI 7.10.5.1-3)
   - #3 ties for #10 and smaller long. bars
   - #4 ties for #11 and larger long. bars
Anti-buckling spacing requirements:

\[ s_{\text{max}} \leq 16 \, d_{b,\text{long}} \]
\[ s_{\text{max}} \leq 48 \, d_{b,\text{tie}} \]
\[ s_{\text{max}} \leq b, h \]

Tie Arrangements

- 4 bars
- 6 bars
- 8 bars
- 10 bars
- 12 bars
(2) Column shear reinforcement

\[ \text{ACI (11.5.5.1): } \text{If } V_{\text{demand}} \geq 0.5 \phi V_c, \]
\[ \text{Then shear rebars req'd} \]

\[ \text{ACI (11.5.4.1): } s_{\text{max}} \leq d/2 \]

(3) Special Requirements for lap splice regions

\[ \text{ACI (7.10.5.2), ACI (7.10.5.5), ACI (7.8.1.3)} \]

(4) Spirals

\[ \text{ACI (Eq. 10.5)} \quad s \leq \prod d_{sp}^2 f_y / \{0.45 D_c f_c' \ [Ag/\text{Ac} - 1]\} \]

\[ \text{ACI (7.10.4.3) for confinement} \]
\[ s \leq 3" \]
ACI (3.3.3) and (7.10.4.3) for congestion
\( s \geq 1 \)

**Slender Column Design**

Refer to ACI (10.11) Moment Magnifier Method

\[
M_{\text{design}} = \delta_b \cdot M_{2b} + \delta_s \cdot M_{2s}
\]

where
\[
\delta_b = \frac{C_m}{(1 - Pu / oPc)} \geq 1.0
\]
\[
\delta_s = \frac{1}{(1 - \Sigma P_u / \phi \Sigma P_c)} \geq 1.0
\]
\[
P_c = \frac{\pi^2 EI}{(kl_u)^2}
\]

For braced frame, if \( kl/r < 34 - 12 \frac{M_{1b}}{M_{2b}} \), then neglect slenderness.
For unbraced frame, if \( kl/r < 22 \), then neglect slenderness
Footings
Structural Action of Strip and Spread Footings

(a) Soil Pressures

\[ q = \frac{P}{A} \pm M y / l \]

Gross Soil Pressure (footing weight + overburden soil + factored loading) must be less or equal to the allowable soil pressure.
Net Soil Pressure (neglect overburden soil, conservative) for design of footing for shear and bending.

(b) Flexure (ACI 15.3 and 15.4.2):

- For square or rectangular columns, critical section at the face of the column

- For circular or regular polygon columns, critical section at face of imaginary square with same area.

- For footing supporting masonry walls, critical section at halfway between middle and edge of wall

- For column with steel base plates, critical section at halfway between column face and edge of base plate

ACI (15.4.3) Flexural reinforcement shall be distributed uniformly across entire width of a square footing
ACI (15.4.4) For rectangular footings, rebar for long direction shall be distributed uniformly across width and portion of rebar for short direction shall be uniformly distributed in center bandwidth and the remaining place outside the bandwidth.

Minimum flexural reinforcement (use slabs, not specified in ACI)

for $f_y = 40$ ksi,  $A_s, \text{min} = 0.0020\, bh$

for $f_y = 60$ ksi,  $A_s, \text{min} = 0.0018\, bh$

Maximum rebar spacing (ACI 7.6.5)

$s_{max} \leq 3h$ and $s_{max} \leq 18''$

Development:

Bar must extend at least $l_d$ from critical section
\[ q = \frac{P}{a \times b} \]
\[ M_{A-A} = q \times b \times f \left( \frac{f}{2} \right) \]

\[ M_{B-B} \text{ is similar} \]
(c) One-Way Shear

\[ \phi ( V_s + V_c ) \geq V_{\text{demand}} \]
\[ V_s = 0 \]
\[ V_c = 2 \sqrt{f_c} \quad b \quad d \]
\[ V_{\text{demand}} = q ( \text{trib. area} ) \]
(d) Two-Way (Punching) Shear

Tributary Area for Two-Way Shear

\[ \phi ( V_s + V_c ) \geq V_{\text{demand}} \]
\[ V_s = 0 \]

(a) \[ V_c = (2 + 4 / \beta ) \sqrt{f_{c'}} bd \]
(b) \[ V_c = (\alpha_s d / \beta + 2 ) \sqrt{f_{c'}} bd \]
(c) \[ V_c = 4 \sqrt{f_{c'}} bd \]

\( \alpha_s = 40 \text{ Int, } 30 \text{ edge, } 20 \text{ corner cols} \)
\( \beta = \text{long side / short side} \)
\( V_{\text{demand}} = q (\text{trib. area}) \)
(e) Allowable column concrete bearing stress

\[ f_{all} = \phi (0.85) \, f_c' \, \sqrt{\frac{A_2}{A_1}} \leq \phi (1.7 \, f_c') \]

(f) Load Transfer from Column to Footing

(ACI 15.8) \( A_{dowel} \geq 0.005 \, A_{g, column} \)

**Standard Practice:** at least four dowel bars are used.

**Generally:** number of dowel bars = number of column bars

**Development:** Dowel bars must be embedded in footing a distance exceeding the development length.
Footing Design Procedure

1. Select footing size to satisfy soil bearing capacity
2. Determine footing effective depth \(d\)
3. Calculate net factored earth pressure
4. Check one-way shear (if inadequate, increase footing depth or width)
5. Check two-way shear (see above)
6. Design for bending
7. Check development length
8. Verify column bearing capacity
9. Design dowels
Ex. 2: Design a square 2-way footing to carry live load of 240 kips which is transmitted through a 16” square column. The allowable soil pressure is 4000 psf and the top of the footing is level with the surrounding soil surface. Use 3000 psi concrete and 40,000 steel. Disregard column dead loading, but include footing dead load.

- Assume footing thickness, \( h_f = 20" \) and \( d = 16" \)

- The unfactored net allowable soil pressure
  \[ p_{a,\text{net}} = 4000 - \left( \frac{20}{12} \right) (150) = 3750 \text{ psf} \]

- Req’d footing area = \( \frac{240,000}{3,750} = 64 \text{ ft}^2 \)
  use 8 ft x 8 ft square footing

- \( p_{\text{act}} = 3,750 \text{ psf} \) (net pressure)
Check One-Way Shear

Critical Area = \((8 * 12) * (48 - 8 - 16)\)
= 2304 in\(^2\) = 16 ft\(^2\)

\(Vu = 1.7 \times 3750 \times 16 = 102,000 \text{ lb (demand)}\)

\(\phi Vc = 0.85 (2) \sqrt{3000} (96) (16) = 143,021 \text{ lb} > Vu \text{ OK!!!!!!} \)

Check Two-Way Shear

Critical Area = \((8 * 12)^2 - (32)^2\)
= 8192 in\(^2\) = 56.9 ft\(^2\)

Critical Perimeter = 4 \times (32/12) = 10.67 ft
\(Vu = 1.7 \times 3750 \times 56.9 = 362,674 \text{ lb (demand)}\)
\[ \beta = \text{long/short} = 8/8 = 1 \]
(a) \[ V_c = (2 + 4 \div 1) \sqrt{3000 \times (10.67 \times 12) \times (16)} = 673,252 \text{ lb} \]
(b) \[ V_c = (40 \times 16/1 + 2) \sqrt{3000 \times (10.67 \times 12) \times (16)} = 7.2 \text{ e 7 lb} \]
(c) \[ V_c = 4 \sqrt{3000 \times (10.67 \times 12) \times (16)} = 448,834 \text{ lb} \leq \text{ governs} \]

\[ \phi V_c = 0.85 \times 448,834 = 381,509 \text{ lb} > V_u = 362,674 \text{ lb} \quad \text{OK} \text{ !!!!} \]

**Design for Bending**

- \( M_u = \frac{1}{2} w L^2 = \frac{1}{2} (1.7 \times 3750) (8) (3.33)^2 = 282,767 \text{ ft-lb} \)

- \( k = 1 - \sqrt{1 - \left(\frac{2M_u}{\phi}\right) \div \left(0.85 \text{ fc'} b d^2\right)} \)
  \[= 1 - \sqrt{1 - \left(2 \times 282767 \times 12 / 1000 / 0.9\right) \div \left(0.85 \times 3000 \times (8 \times 12) \times 16^2\right)} \]
  \[= 0.0621 \]

- \( 0.75 k_{bal} = 0.75 \times 0.85 \times (87 / (87+40)) = 0.436 \quad \text{OK} \text{ !!!!} \)

- \( A_s, \text{ req'd} = \frac{M_u}{\phi} \div [f_y d \times (1 - k/2)] \)
  \[= \left(282.767 \times 12 / 0.9\right) \div [40 \times 16 \times (1 - 0.0621/2)] = 6.08 \text{ in}^2 \]
- Use 11#7 bars (As, prov = 6.6 in\(^2\)) or #7 bars @ 9" centers ≤ 3*h or 18"

- Check development:

\[
l_d = 0.04 \frac{A_b \, f_y}{\sqrt{f_{c'}}} = 0.04 \times 0.60 \times 40000 / \sqrt{3000} = 17.5"
\geq 12"

Available = 4*12 - 8 = 40"  OK!!!!!!

[Diagram showing #7 bars at 9" centers]
Prestressed Concrete

ACI Provisions for Stress (ACI 18.4 and 18.5)

- Max. tensile stress in tendon prior to transfer of stress to the concrete:
  \[ f_t = \min \{ 0.94 f_{py} \text{ or } 0.80 f_{pu} \} \]

- Immediately after the prestress is transferred to the concrete, the max tensile stress in the tendon is:
  \[ f_t = \min \{ 0.82 f_{py} \text{ or } 0.74 f_{pu} \} \]

- In post-tensioned construction, the max tendon tensile stress immediately after tendon anchorage is:
  \[ f_t = 0.70 f_{pu} \]

- The max. extreme fiber compressive stress in the concrete immediately after prestress transfer is:
  \[ f_{c'} = 0.60 f_{ci} \]

where \( f_{ci} = \) compressive strength of concrete at that time
- The max extreme fiber tensile stress in the concrete immediately after prestress transfer is:

\[ f_t = 3 \sqrt{f_{ci}} \]

*for tension at the ends of simply supported member, use 2 \( f_t \)

- If tensile stresses in the concrete are exceeded, provide additional reinforcement in the tension zone.

- At service load and after all prestress losses, the max extreme fiber compression and tension stresses in the concrete are:

\[ f_c = 0.45 \ f_{ci} \]

\[ f_t = 6 \sqrt{f_{ci}} \]
Analysis of Prestressed Simply Supported Beams w/ Uniform Load

1. Estimate dead weight per foot of beam, \( w_d \)

2. Determine max moment on beam from dead weight
   \[ M_d = w_d \cdot \frac{L_2}{8} \]

3. Assume full beam in compression and calculate NA based on transformed area.

4. Determine moment of inertia (I) about NA for transformed area

5. Calculate eccentricity \( e \) of the strands

6. Calculate stresses due to prestressing force \( P \)
   \[ f_{pc} = \frac{-P}{A} \pm \frac{P \cdot e \cdot c}{I} \]

7. Calculate stresses due to dead weight
   \[ f_d = \frac{+M_d c}{I} \]
8. Stress distribution immediately after transfer
   \[ f = f_{pc} + f_d \]

9. Check to see if tensile stresses in concrete are below max allowed

10. Check to see if compressive stresses in concrete are below max. allowed

11. Calculate stress distribution due to live load
    \[ f_l = \frac{M_l}{c/l} \]

12. Calculate service load stress distribution
    \[ f_{total} = f_{pc} + f_d + f_l \]

13. Check long term stresses
ASTM Standard Prestressing Tendons

(strand grade is $f_{pu}$ in ksi)

<table>
<thead>
<tr>
<th>type</th>
<th>nominal diameter, in.</th>
<th>nominal area, sq in.</th>
<th>nominal weight, lb per ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>grade 250</td>
<td>1/4 (0.250)</td>
<td>0.036</td>
<td>0.12</td>
</tr>
<tr>
<td>seven wire</td>
<td>5/16 (0.313)</td>
<td>0.058</td>
<td>0.20</td>
</tr>
<tr>
<td>strand</td>
<td>3/8 (0.375)</td>
<td>0.108</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>1/2 (0.500)</td>
<td>0.144</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(0.600)</td>
<td>0.216</td>
<td>0.74</td>
</tr>
<tr>
<td>grade 270</td>
<td>3/8 (0.375)</td>
<td>0.085</td>
<td>0.29</td>
</tr>
<tr>
<td>seven-wire</td>
<td>7/16 (0.438)</td>
<td>0.115</td>
<td>0.40</td>
</tr>
<tr>
<td>strand</td>
<td>1/2 (0.500)</td>
<td>0.153</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>(0.600)</td>
<td>0.215</td>
<td>0.74</td>
</tr>
<tr>
<td>prestressing wire</td>
<td>0.192</td>
<td>0.029</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>0.196</td>
<td>0.030</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.250</td>
<td>0.049</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>0.276</td>
<td>0.060</td>
<td>0.20</td>
</tr>
<tr>
<td>smooth</td>
<td>3/4</td>
<td>0.44</td>
<td>1.50</td>
</tr>
<tr>
<td>prestressing bars</td>
<td>7/8</td>
<td>0.60</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.78</td>
<td>2.67</td>
</tr>
<tr>
<td></td>
<td>1-1/8</td>
<td>0.99</td>
<td>3.38</td>
</tr>
<tr>
<td></td>
<td>1-1/4</td>
<td>1.23</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>1-3/8</td>
<td>1.48</td>
<td>5.05</td>
</tr>
<tr>
<td>deformed</td>
<td>5/8</td>
<td>0.28</td>
<td>0.98</td>
</tr>
<tr>
<td>prestressing bars</td>
<td>3/4</td>
<td>0.42</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.85</td>
<td>3.01</td>
</tr>
<tr>
<td></td>
<td>1-1/4</td>
<td>1.25</td>
<td>4.39</td>
</tr>
<tr>
<td></td>
<td>1-3/8</td>
<td>1.56</td>
<td>5.56</td>
</tr>
</tbody>
</table>
Properties and Design of Masonry (ASD)

**Defn.** Built-up construction of masonry units bonded together with mortar and filled with grout.

**Basic Components:** masonry units (clay and concrete); mortar; grout; and reinforcement.

*Figure 3-17: Components of a typical masonry wall.*
Types of Masonry

(a) Clay Masonry (brick):

- Solid brick - holes make up < 25% of gross brick area
- Hollow brick - holes make up > 25% and < 60% of gross brick area

Figure 3-18: Examples of commonly used clay masonry units.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t$</td>
<td>$h$</td>
<td>$l$</td>
</tr>
<tr>
<td>Standard Modular</td>
<td>4</td>
<td>2-2/3</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>Engineer</td>
<td>4</td>
<td>3-1/5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>Economy</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/2</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-6: Nominal and manufactured dimensions for clay masonry units.

<table>
<thead>
<tr>
<th>Unit Designation</th>
<th>Thickness in.</th>
<th>Face Dimension</th>
<th>Number of Courses in 16 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Height, in.</td>
<td>Length, in.</td>
</tr>
<tr>
<td>Standard Modular</td>
<td>4</td>
<td>2-2/3</td>
<td>8</td>
</tr>
<tr>
<td>Engineer</td>
<td>4</td>
<td>3-1/5</td>
<td>8</td>
</tr>
<tr>
<td>Economy</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3-7: Modular units for clay masonry.
(b) Concrete Masonry (concrete block)

Figure 3-20: Typical concrete masonry unit.

<table>
<thead>
<tr>
<th>Type of Block</th>
<th>Net Area Strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular-strength block</td>
<td>2,000</td>
</tr>
<tr>
<td>High-strength block</td>
<td>3,500</td>
</tr>
<tr>
<td>Extra-high-strength block</td>
<td>5,000</td>
</tr>
</tbody>
</table>

Table 3-8: Net area compressive strengths for concrete masonry units.
(c) **Mortar and Grout** - used to bond masonry units. Mortar is used as a bedding material in the joints between the units and grout is used to fill voids in the units.

(d) **Steel Reinforcement** - Properties are same as used in concrete

**For allowable stress design,**
- The tensile stress in the reinforcement shall not exceed:
  (i) Grade 40 or 50 steel   20 ksi
  (ii) Grade 60 steel        24 ksi
  (iii) Wire Reinforcement   30 ksi

- The compressive stress in the reinforcement shall not exceed:
  \[0.4 \, f_y \leq 24 \, \text{ksi}\]
### Masonry Behavior

(a) $f_m'$

*UBC Table 24-C  Specified Compressive Strength of Masonry, $f_m$*

<table>
<thead>
<tr>
<th>Compressive Strength of Clay Masonry Units (psi)</th>
<th>Specified Compressive Strength of Masonry, $f_m$, (psi)</th>
<th>Type M or S Mortar</th>
<th>Type N Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>14,000 or more</td>
<td>5,300</td>
<td>4,400</td>
<td></td>
</tr>
<tr>
<td>12,000</td>
<td>4,700</td>
<td>3,800</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>4,000</td>
<td>3,300</td>
<td></td>
</tr>
<tr>
<td>8,000</td>
<td>3,350</td>
<td>2,700</td>
<td></td>
</tr>
<tr>
<td>6,000</td>
<td>2,700</td>
<td>2,200</td>
<td></td>
</tr>
<tr>
<td>4,000</td>
<td>2,000</td>
<td>1,600</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compressive Strength of Concrete Masonry Units (psi)</th>
<th>Type M or S Mortar</th>
<th>Type N Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,800 or more</td>
<td>3,000</td>
<td>2,800</td>
</tr>
<tr>
<td>3,750</td>
<td>2,500</td>
<td>2,350</td>
</tr>
<tr>
<td>2,800</td>
<td>2,000</td>
<td>1,850</td>
</tr>
<tr>
<td>1,900</td>
<td>1,500</td>
<td>1,350</td>
</tr>
<tr>
<td>1,250</td>
<td>1,000</td>
<td>950</td>
</tr>
</tbody>
</table>
(b) \( E_m = \) Modulus of Elasticity

\[ E_v \ (G) = \text{Shear Modulus} = 0.4 \ E_m \]

### Table 5.5.1.2 – Clay Masonry

<table>
<thead>
<tr>
<th>Compressive Strength of Units, psi</th>
<th>( E_m ) psi ( \times 10^6 )</th>
<th>Type N Mortar</th>
<th>Type S Mortar</th>
<th>Type M Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 12,000</td>
<td>2.8</td>
<td>3.0</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>2.4</td>
<td>2.9</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>8000</td>
<td>2.0</td>
<td>2.4</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>1.6</td>
<td>1.9</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.8</td>
<td>0.9</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.5.1.3 – Concrete Masonry

<table>
<thead>
<tr>
<th>Compressive Strength of Units, psi</th>
<th>( E_m ) psi ( \times 10^6 )</th>
<th>Type N Mortar</th>
<th>Type S or M Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 6000</td>
<td></td>
<td>-</td>
<td>3.5</td>
</tr>
<tr>
<td>5000</td>
<td></td>
<td>2.8</td>
<td>3.2</td>
</tr>
<tr>
<td>4000</td>
<td></td>
<td>2.6</td>
<td>2.9</td>
</tr>
<tr>
<td>3000</td>
<td></td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td>2500</td>
<td></td>
<td>2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>1.8</td>
<td>2.2</td>
</tr>
<tr>
<td>1500</td>
<td></td>
<td>1.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>
## Section Properties

### (a) Concrete Masonry Units

| Nominal Block Thickness, in. | Net Area, $A_n$, in² | Moment of Inertia, $I$, in⁴ | Section Modulus, $S$ (per ft. of length) for: | | | | fully bedded | | | | face shell bedded | | | | 4 | 18 | 38 | 21 | 28 | 45 | 25 | 6 | 24 | 130 | 46 | 37 | 139 | 50 | 3 | 30 | 309 | 81 | 48 | 334 | 88 | 10 | 33 | 567 | 118 | 60 | 634 | 132 | 12 | 36 | 929 | 160 | 68 | 1063 | 183 |

Table 3-14: Section properties for concrete masonry units.

### (b) Clay Masonry Units - Use net area of cross section
# Allowable Working Stresses

## Special Inspection Required

<table>
<thead>
<tr>
<th>Types of Stress</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Compression, axial: Walls</td>
<td>$F_a = 0.20 f'_m R^*$</td>
<td></td>
</tr>
<tr>
<td>2. Compression, axial: Columns</td>
<td>$P_a = (0.20 f'<em>m A_s + 0.65 A</em>{st} F_{sc}) R^*$</td>
<td>$F_a = P_a / A_s$</td>
</tr>
<tr>
<td>3. Compression, flexural</td>
<td>$F_b = 0.33 f'_m$, 2,000 psi maximum</td>
<td></td>
</tr>
<tr>
<td>4. Shear: (flexural members)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. No shear reinforcement, masonry taking all shear</td>
<td>$F_v = 1.0 \sqrt{f'_m}$, 50 psi maximum</td>
<td></td>
</tr>
<tr>
<td>b. Reinforcing taking all shear</td>
<td>$F_v = 3.0 \sqrt{f'_m}$, 150 psi maximum</td>
<td></td>
</tr>
</tbody>
</table>
Shear: (shear walls)  

For $M/Vd < 1$,

$$F_v = \frac{\gamma}{2} \left( 4 - \frac{M}{Vd} \right) \sqrt{f'_{m}},$$

$$\left( 80 - 45 \frac{M}{Vd} \right)$$ psi maximum

For $M/Vd > 1$,

$$F_v = 1.0 \sqrt{f'_{m}},$$

35 psi maximum

d. Shear reinforcement designed to take all shear

For $M/Vd < 1$,

$$F_v = \frac{\gamma}{2} \left( 4 - \frac{M}{Vd} \right) \sqrt{f'_{m}},$$

$$\left( 120 - 45 \frac{M}{Vd} \right)$$ psi maximum

For $M/Vd > 1$,

$$F_v = 1.5 \sqrt{f'_{m}},$$

75 psi maximum

5. Modulus of Elasticity

$$E_m = 750 f'_{m}, \max 3,000,000 \text{ psi}$$

6. Modulus of Rigidity

$$G = 0.4 E_m$$

7. Bearing on Full Area

$$F_{br} = 0.26 f'_{m}$$

8. Bearing on One-Third Area or Less

$$F_{br} = 0.38 f'_{m}$$

9. Bond-Plain Bars

$$u = 60$$

10. Bond-Deformed Bars

$$u = 200$$

$$R = 1 - \left( \frac{H^2}{42t} \right)$$

$$u = kL$$ (Effective height)

1. Exception: For a distance of one-sixteenth the clear span beyond the point of collection stress shall be 20 psi.

2. $M$ is the maximum bending moment occurring simultaneously with the shear load $V$ at the section under consideration.

3. This increase applies only when the least distance between the edges of the loaded and unloaded areas is a minimum of one-fourth of the parallel side dimension of the loaded area. The allowable bearing stresses on a reasonably concentric area greater than one-third but less than the full area shall be interpolated linearly.